EFFECT OF STEEPNESS OF RISE AND FALL OF THE INPUT PULSE ON THE RESPONSE OF PULSE AMPLIFIERS (PART II)*

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ABSTRACT. A study of the response characteristics of shunt-compensated amplifiers has been made for a ramp function input. The magnitude of the decreasing peaks of the highly damped overshoot oscillation has been determined. It has been shown that the amplitude of the peak overshoot may be made insignificant for many applications when the rise time of the incoming wave-front is several times the RC time-constant of the plate circuit of the amplifier. In this case the contribution of the amplifier to the rise and delay times of the output pulse has also been found to be negligible and it has been observed that the output may be sharper compared to the input for values of $m=(L/CR^2)$ higher than a lower limit which is determined by the rise time of the incoming pulse.

The periods of overshoot oscillations and the reduction factor of the successive overshoot peaks have been given in the form of a table for all the interesting values of m. In the cases of pulses having sharp rise and fall, expressions for the maximum output voltage obtainable have been derived. The peaks of undershoots have also been determined.

INTRODUCTION

In a previous communication (Bhattacharyya, 1954) a study of the effect of steepness of rise and fall of the input pulse on the response characteristics of an RC-coupled pulse amplifier was made. It was observed that the two important figures of merit of such an amplifier, e.g. rise and delay times of the transmitted pulse, are markedly dependent on the build-up time of the input waveform.

This paper presents a detailed study of the response characteristics of a shunt-compensated amplifier to pulses of the following types: (*i*) a ramp function input, (*ii*) a pulse with linear rise and fall and (*iii*) a saw-tooth pulse. In this type of amplifier the advantage of decrease in rise and delay times is somewhat offset due to the appearance of over-shoots and overshoot oscillations in the case of a ramp function input. The resistances in series with the coil that is used for high-frequency compensation make

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the overshoot oscillations highly damped. The decreasing peaks of such oscillations and the times corresponding to the peaks have been determined.

In the steady state analysis of such amplifiers stress is generally laid on achieving constant gain and linear phase relations over the utilised range of frequency. When the interest is switched over to the determination of transient characteristics, the main object in the design lies in obtaining a monotonically increasing response. But it is not possible to attain such a response with two or four-pole coupling networks consisting of all the three linear circuit elements, e.g. resistances and inductors and capacitors.

The influence of the circuit parameter m, which is a dimensionless quantity defined as the ratio of inductance L and CR^2, on the response has also been studied in order to choose a suitable value of m. It is known that by an increase in m we obtain a significant improvement in the rise time of the amplifier with a consequent increase in the magnitude of overshoots and overshoot oscillations. So the value of m should be chosen with the object of having a reduced figure of both the rise time and the peak of the overshoot oscillation. This paper attempts to furnish all the necessary informations regarding the nature of the response for all the widely used values of m. In the cases of pulses having sharp rise and fall, expressions for the maximum output voltage obtainable and the peaks of undershoots have been derived.

RESPONSE TO A RAMP FUNCTION INPUT PULSE

A typical circuit diagram of a shunt-compensated amplifier is given in figure 1. Only the high frequency equivalent circuit (figure 2) of the amplifier will be considered. With the aid of this figure the expression for the output voltage may be written as :

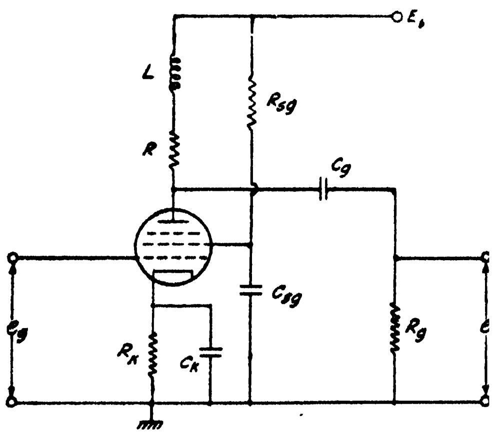

FIG. 1. Circuit diagram of a shunt-compensated amplifier

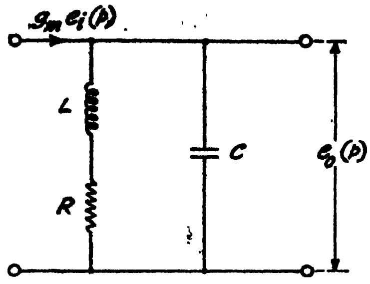

FIG. 2 High frequency equivalent circuit of a shunt-compensated amplifier.

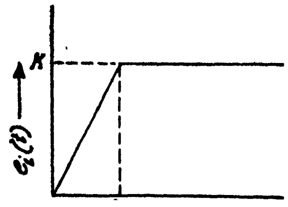

FIG. 3. A ramp function input pulse

$$e_0(p)=g_m e_i(p).\ \frac{R+pL}{p^2LC+pRC+1} \quad \ldots \quad (1)$$

where

$e_i(p)$ = Laplace transform of the input excitation voltage,

g_m = mutual transconductance of the tube,

R = equivalent resistance in series with the coil,

L = inductance of the coil.

and C = stray and wiring capacitances across the coil.

Since the input pulse is a ramp-fuuction (figure 3) we obtain (Bhattacharyya, 1954) :

$$e_i(p)=\frac{1}{t_1}.\ \frac{1-\epsilon^{-pt_1}}{p^2} \quad (2)$$

where t_1 is the build-up time of the input.

Substituting (2) in (1), we have

$$e_0(p)=\frac{g_m}{t_1}.\ \frac{1-\epsilon^{-pt_1}}{p^2}.\ \frac{R+pL}{p^2LC+pRC+1} \quad (3)$$

Now introducing the circuit parameter

$$m=L/CR^2 \quad (4)$$

in (3), we shall obtain,

$$e_0(p)=\frac{g_mR}{t_1}.\ \frac{1-\epsilon^{-pt_1}}{p^2}.\ \frac{1+(pCR).\ m}{(pCR)^2.m+(pCR)+1} \quad (5)$$

Normalizing (5) by the substitution $t=t/CR$ and $e_0(t)=\dfrac{e_0(t)}{g_mR}$, we have

$$e_0(p)=\frac{1-\epsilon^{-pt_r}}{t_r}.\ \frac{1+mp}{p^2(mp^2+p+1)} \quad \ldots \quad (6)$$

where $t_r=t_1/RC$

Equation (6) may be expressed as

$$e_0(p)=\frac{1-\epsilon^{-pt_r}}{t_r}.\ \frac{(p+a_1)}{p^2(p+a_2)(p+a_3)} \quad \ldots \quad (7)$$

where $a_1 = 1/m$, $a_2 = (1/2m).(1+\sqrt{1-4m})$ and $a_3 = (1/2m).(1-\sqrt{1-4m})$

Taking the inverse Laplace transform of (7), we have

$$e_0(t) = \frac{1}{t_r}.\left[\frac{(a_1-a_2)}{a_2^2(a_3-a_2)}.\epsilon^{-a_2 t} + \frac{(a_1-a_3)}{a_3^2(a_2-a_3)}\epsilon^{-a_3 t} + \frac{a_1}{a_2 a_3}t + \frac{a_2 a_3 - a_1(a_2+a_3)}{a_2^2 a_3^2}\right] - \frac{1}{t_r}.\left[\frac{(a_1-a_2)}{a_2^2(a_3-a_2)}.\epsilon^{-a_2(t-t_r)} + \frac{(a_1-a_3)}{a_3^2(a_2-a_3)}.\epsilon^{-a_3(t-t_r)} + \frac{a_1}{a_2 a_3}(t-t_r) + \frac{a_2 a_3 - a_1(a_2+a_3)}{a_2^2 a_3^2}\right].u(t-t_r) \quad \dots (8)$$

With the help of equation (8), the equations for the transient response may be written in the following way :

$$e_0(t) = \frac{1}{t_r}.\left[\frac{(a_1-a_2)}{a_2^2(a_3-a_2)}\epsilon^{-a_2 t} + \frac{(a_1-a_3)}{a_3^2(a_2-a_3)}\epsilon^{-a_3 t} + \frac{a_1}{a_2 a_3}t + \frac{a_2 a_3 - a_1(a_2+a_3)}{a_2^2 a_3^2}\right] \quad (0 \leq t \leq t_r) \quad \dots (9)$$

and

$$e_0(t) = \frac{1}{t_r}.\left[\frac{(a_1-a_2)}{a_2^2(a_3-a_2)}(1-\epsilon^{a_2 t_r})\epsilon^{-a_2 t} + \frac{(a_1-a_3)}{a_3^2(a_2-a_3)}.(1-\epsilon^{a_3 t_r})\epsilon^{-a_3 t}\right] + \frac{a_1}{a_2 a_3}, \quad (t \geq t_r) \quad \dots (10)$$

Both the equations (9) and (10) lead to an identical result at the time $t = t_r$.

We shall now consider three special cases, e.g. (*i*) $m < \frac{1}{4}$, (*ii*) $m = \frac{1}{4}$ and (*iii*) $m > \frac{1}{4}$. For the first case we obtain from equations (9) and (10):

$$e_0(t) = \frac{1}{t_r}.\frac{m^2}{\sqrt{1-4m}}.\epsilon^{-t/2m}.\left[\frac{1+\sqrt{1-4m}}{1-2m-\sqrt{1-4m}}.\epsilon^{\sqrt{1-4m}.t/2m} - \frac{1-\sqrt{1-4m}}{1-2m+\sqrt{1-4m}}.\epsilon^{\sqrt{1-4m}.t/2m}\right] + \frac{1}{t_r}(t+m-1), \quad (0 \leq t \leq t_r) \quad (11)$$

and

$$e_0(t) = \frac{1}{t_r}\left[A_1\epsilon^{-(1-\sqrt{1-4m})t/2m} - A_2\epsilon^{-(1+\sqrt{1-4m})t/2m}\right] + 1, \quad (t \geq t_r) \quad (12)$$

where

$$A_1 = \frac{m^2(1+\sqrt{1-4m})}{\sqrt{1-4m}(1-2m-\sqrt{1-4m})}.\left\{1-\epsilon^{(1-\sqrt{1-4m})t_r/2m}\right\}$$

and

$$A_2 = \frac{m^2(1-\sqrt{1-4m})}{\sqrt{1-4m}(1-2m+\sqrt{1-4m})}.\left\{1-\epsilon^{(1+\sqrt{1-4m})t/2m}\right\}$$

For case (*ii*) $(m=\frac{1}{4})$, the equations for the transient response may be given in the following form :

$$e_0(t)=\frac{2m}{t_r}\left[(t+6m)\epsilon^{-t/2m}+2(t-3m)\right],\ (0\leq t\leq t_r) \tag{13}$$

and

$$e_0(t)=\frac{2m}{t_r}\left\{(t+6m)\epsilon^{-t/2m}(1-\epsilon^{t_r/2m})\right\}+2m\epsilon^{-(t-t_r)/2m}+1,\ (t\geq t_r) \quad \ldots \tag{14}$$

We shall now consider the last and the most important case $(m>\frac{1}{4})$ in which the constants a_1, a_2 and a_3 are given

$$a_1=\frac{1}{m},\ a_2=\frac{1}{2m}(1+j\sqrt{4m-1})\text{ and }a_3=\frac{1}{2m}(1-j\sqrt{4m-1}).$$

Substituting these values in equations (9) and (10) and simplifying, we obtain

$$e_0(t)=\frac{1}{t_r}\cdot\frac{\epsilon^{-t/2m}}{\sqrt{4m-1}}\Bigg[(1-m)\sqrt{4m-1}\cos\left(\frac{\sqrt{4m-1}}{2m}t\right)$$
$$+(1-3m)\sin\left(\frac{\sqrt{4m-1}}{2m}t\right)\Bigg]+\frac{1}{t_r}(t+m-1),\ (0\leq t\leq t_r) \tag{15}$$

and

$$e_0(t)=\frac{1}{t_r}\cdot\frac{\epsilon^{-t/2m}}{\sqrt{4m-1}}\left[A\cos\left(\frac{\sqrt{4m-1}}{2m}t\right)+B\sin\left(\frac{\sqrt{4m-1}}{2m}t\right)\right]+1,\ \geq t_r \tag{16}$$

where

$$A=(1-m)\sqrt{4m-1}\left[1-\epsilon^{t_r/2m}\cos\left(\frac{\sqrt{4m-1}}{2m}t_r\right)\right]$$
$$+(1-3m)\left[\epsilon^{t_r/2m}\sin\left(\frac{\sqrt{4m-1}}{2m}t_r\right)\right] \tag{17}$$

and

$$B=(1-3m)\left[1-\epsilon^{t_r/2m}\cos\left(\frac{\sqrt{4m-1}}{2m}t_r\right)\right]$$
$$-\epsilon^{t_r/2m}(1-m)\sqrt{4m-1}\sin\left(\frac{\sqrt{4m-1}}{2m}t_r\right) \tag{18}$$

The object of this investigation is to form a correct idea about the reproduction of the sharp leading edges with shunt-compensated amplifiers using practical values of m. The values of m chosen are given below :

(*i*) $m=0.1$, (*ii*) $m=0.2$, (*iii*) $m=0.35$, (*iv*) $m=0.41$,
(*v*) $m=0.50$, (*vi*) $m=0.60$ and (*vii*) $m=1.00$.

The response characteristics have been plotted in figures 4-8 with the aid of equations (11), (12), (15) and (16) for various values of t_r. A plot of the step-function response of such an amplifier is given in Fig. 9. The response function is given by the following expression (Goldman, 1949) :

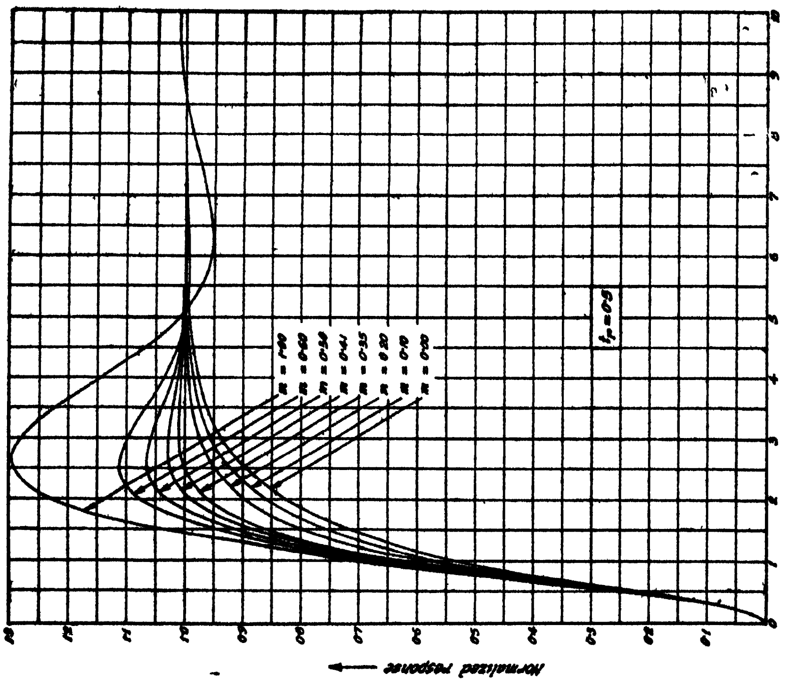

Fig. 5—Response of a shunt-compensated amplifier to a ramp-function input ($t_r = 0.5$)

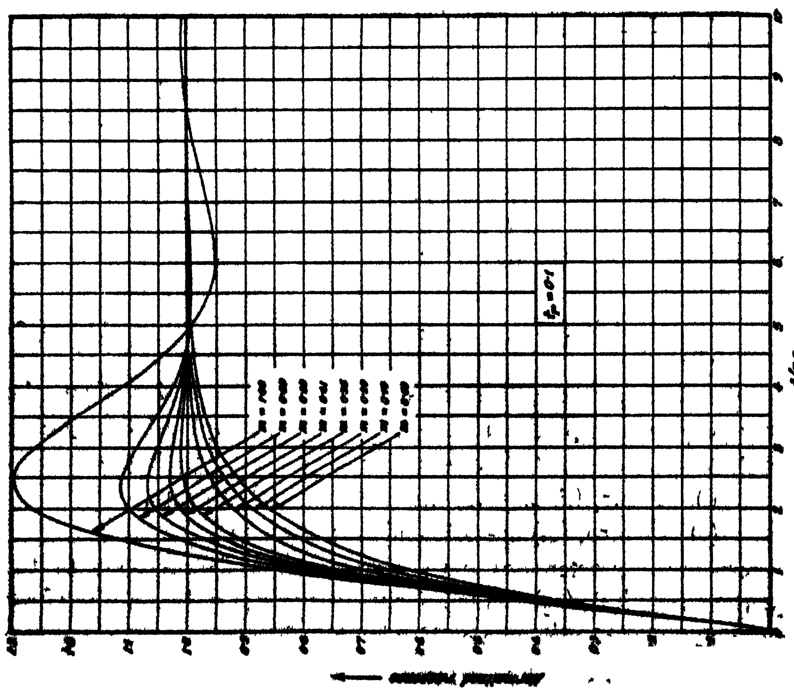

Fig. 4—Response of a shunt-compensated amplifier to a ramp-function input ($t_r = 0.1$).

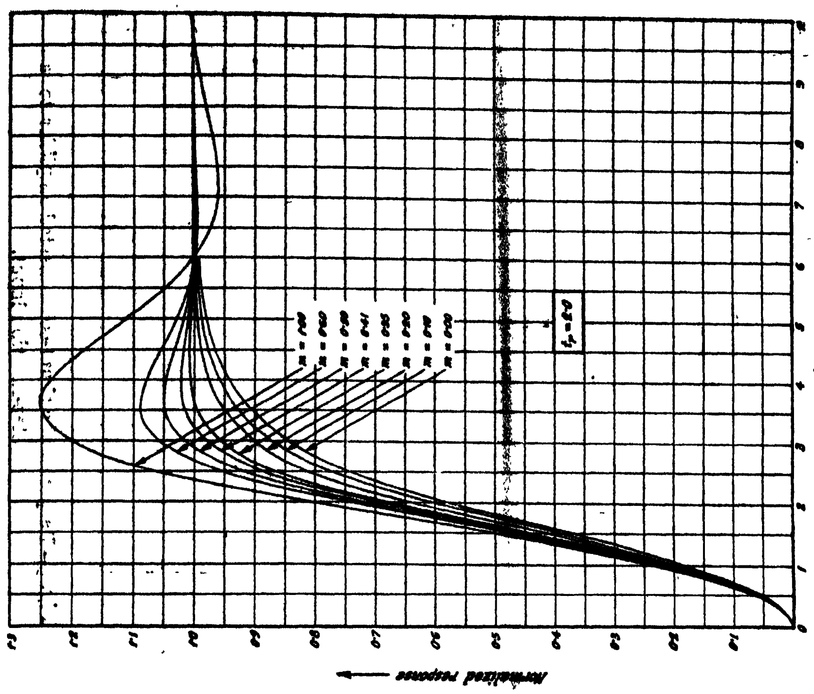

Fig. 7—Response of a shunt-compensated amplifier to ramp-function input (t_r = 2.0).

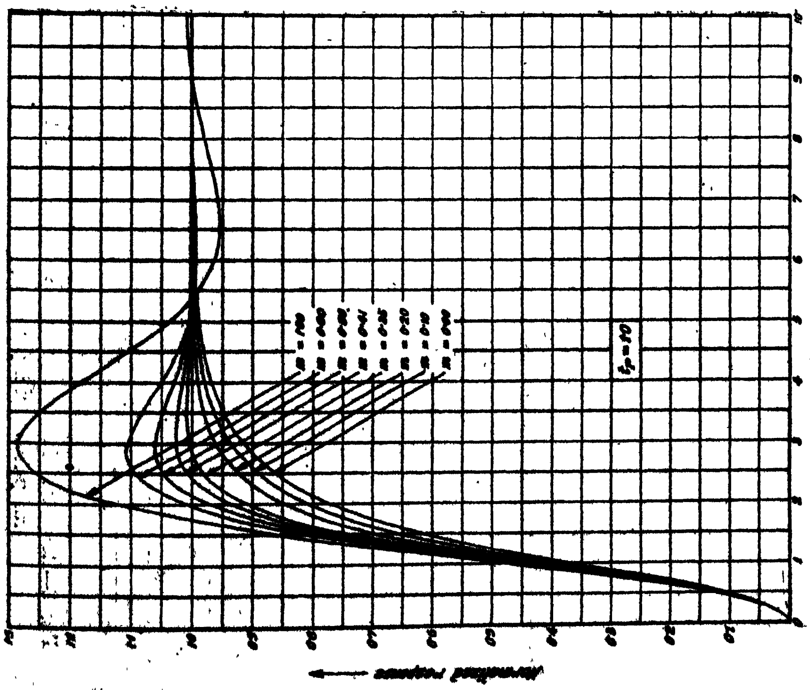

Fig. 6—Response of a shunt-compensated amplifier to a ramp-function input (t_r = 1.0).

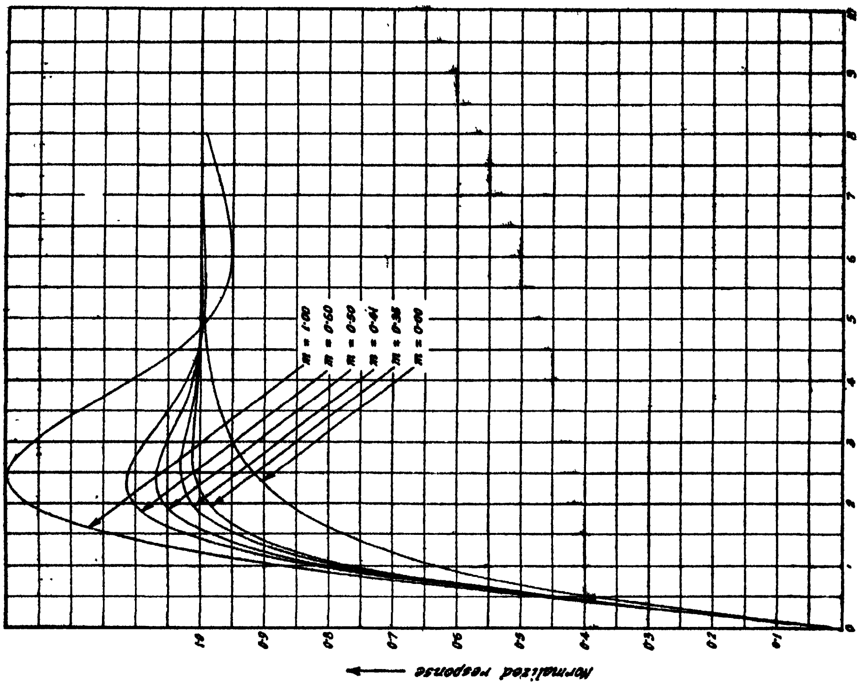

Fig. 9—Response of a shunt-compensated amplifier step-function input.

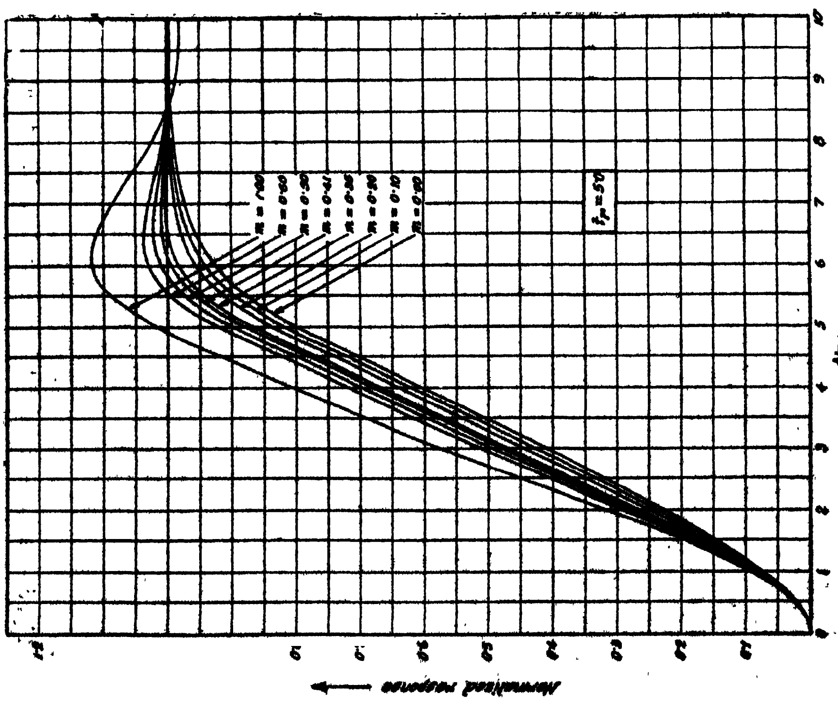

Fig. 8—Response of a shunt-compensated amplifier to a ramp-function input ($t_r = 5.0$.)

$$e_0(t) = 1 - \epsilon^{t/2m} \cdot \left[\cos\left(\frac{\sqrt{4m-1}}{2m}t\right) + \frac{(1-2m)}{\sqrt{4m-1}} \sin\left(\frac{\sqrt{4m-1}}{2m}t\right)\right],$$

$$(m > 0.25)$$

Since the step-function response of a shunt-compensated amplifier has been determined by many authors, we have not here considered the other two cases; e.g. (*i*) $m < 0.25$ and (*ii*) $m = 0.25$. For the sake of comparison, the responses of an ordinary RC-coupled amplifier have been given in every figure.

Characteristics of over-shoot oscillations :—A judicious choice of the circuit parameter m to obtain a well-shaped response with full details of the input requires a previous information regarding the nature of overshoot oscillations. With this purpose in view this section presents the formulae for computing the times of occurrences and the magnitudes of overshoots for various values of m. For $m \leq 0.25$, the nature of response is very much akin to that of an RC-coupled amplifier excepting for slight improvement in rise and delay times. This is why we have considered only the values of m greater than 0.25. The overshoot oscillation generally starts at a time $t > t_r$. With the help of (16) we, then, obtain

$$\frac{de_0(t)}{dt} = \frac{\epsilon^{-t/2m}}{t_r} \left[\left\{ \frac{B}{2m} - \frac{A}{2m\sqrt{4m-1}} \right\} \cos \frac{\sqrt{4m-1}}{2m}t \right.$$

$$\left. - \left\{ \frac{A}{2m} + \frac{B}{2m\sqrt{4m-1}} \right\} \sin \left(\frac{\sqrt{4m-1}}{2m}t \right) \right]. \qquad (19)$$

At the peaks of overshoot oscillation, either positive or negative, the slope of the response function will be zero. Hence, we have from (19) : either

$$\epsilon^{-t/2m} = 0 \qquad (20)$$

or

$$\left\{ \frac{B}{2m} - \frac{A}{2m\sqrt{4m-1}} \right\} \cos \left(\frac{\sqrt{4m-1}}{2m}t \right) - \left\{ \frac{A}{2m} + \frac{B}{2m\sqrt{4m-1}} \right\}$$

$$\sin \left(\frac{\sqrt{4m-1}}{2m}t \right) = 0 \qquad (21)$$

By solving (20) or (21) we shall obtain the times corresponding to the maximum or minimum points of oscillation. It is evident that equation (20) cannot give any practicable solution. We can express equation (21) in the following way :

$$A_0 \cos(\theta t + \phi) = 0 \qquad \ldots \quad (22)$$

where

$$A_0 \cos \phi = \frac{B}{2m} - \frac{A}{2m\sqrt{4m-1}}$$

$$A_0 \sin \phi = \frac{A}{2m} + \frac{B}{2m\sqrt{4m-1}}$$

$$\theta=\frac{\sqrt{4m-1}}{2m} \text{ and } \tan\phi=\frac{B+A\sqrt{4m-1}}{B\sqrt{4m-1}-A}.$$

Solving (22), we obtain the times t_0 corresponding to the peaks of overshoot oscillation. The solution is

$$t_0=\frac{(n+\frac{1}{2})\pi-\phi}{\theta} \qquad \dots (23)$$

where n is an integer having values 0, 1, 2, ... n. Differentiating (19) again we get

$$\frac{d^2e_0(t)}{dt^2}=-\frac{\epsilon^{-t/2m}}{t_r}\left[\frac{A_0}{2m}\cos(\theta t+\phi)+A_0\theta\sin(\theta t+\phi)\right] \qquad \dots (24)$$

At the time t_0, this expression reduces to

$$\frac{d^2e_0(t)}{dt^2}=-\frac{\epsilon^{-t_0/2m}}{t_r}\cdot A_0\theta\sin(\theta t_0+\phi) \qquad \dots (25)$$

An examination of (25) leads us to the conclusion that the peak overshoot points given by (23) correspond to the positive peaks when n is even and to the negative when n is odd.

Fig. 10 shows a typical nature of the overshoot oscillation in which t_{01}, t_{03},...are the times of occurrences of positive peaks,

and t_{02}, t_{04},..., times of occurrences of negative peaks.

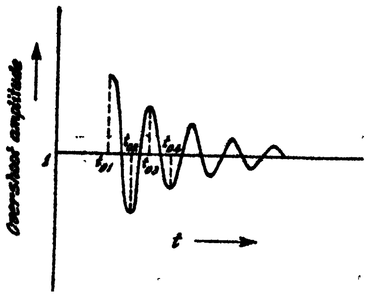

Fig. 10. Nature of the overshoot oscillation.

With the help of (23), we can now set up a relation amongst t_{01}, t_{02}, t_{03}, t_{04}, etc. We can write

$$\left.\begin{aligned}t_{02}&=t_{01}+\pi/\theta,\\ t_{03}&=t_{02}+\pi/\theta,=t_{01}+2\pi/\theta\\ t_{04}&=t_{01}+3\pi/\theta \text{ and so on.}\end{aligned}\right\} \qquad \dots (26)$$

So the period of overshoot oscillation is given by $2\pi/\theta$ which is independent of the rise time of the input pulse.

With the aid of (16), the overshoot amplitude at the time $t=t_{01}$ may be expressed in the following way:

$$O(t_{01})=\frac{1}{t_r}\cdot\frac{\epsilon^{-t_{01}/2m}}{\sqrt{4m-1}}\left[A\cos\theta t_{01}+B\sin\theta t_{01}\right] \quad \ldots (27)$$

At the time $t=t_{02}$, the overshoot amplitude may be written as

$$O(t_{02})=-\frac{1}{t_r}\cdot\frac{\epsilon^{-t_{02}/2m}}{\sqrt{4m-1}}\left[A\cos\theta t_{01}+B\sin\theta t_{01}\right] \quad \ldots (28)$$

since,

$$\theta t_{02}=\pi+\theta t_{01}.$$

Therefore, we have

$$R=\frac{O(t_{02})}{O(t_{01})}=-\epsilon^{-\pi/2\theta m} \quad \ldots (29)$$

where R deno tes the reduction factor of the successive peaks of overshoot oscillations.

Similarly,

$$\frac{O(t_{03})}{O(t_{01})}=\epsilon^{-\pi/\theta m}=R^2 \quad \ldots (30)$$

Equations (29) and (30) give the ratios of attenuation between the successive positive and negative peaks. The periods of overshoot oscillations and the reductio ı factors for all the important values of m are given in Table I. This table will give an idea about the sharp decrease in successive overshoot amplitudes.

TABLE I

m	Period of oscillation	Reduction factor R	R^2
0.35	6.957008	−0.006948	.000048
0.41	6 442860	−0 019672	.000388
0 50	6 285714	−0 043160	001863
0.60	6.3"4878	−0.070215	.004930
1.00	7.258120	−0 164915	.026541

With the help of equations (23), (26), (27), (29) and (30) we have plotted the times reckoning the peaks of overshoot oscillations and the amplitudes of the overshoots in figures 11 and 12.

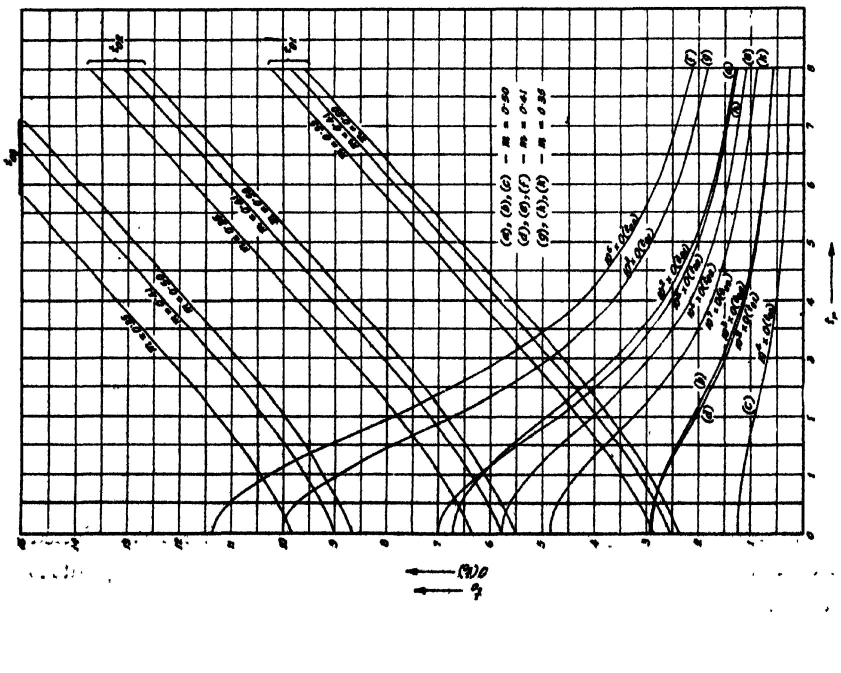

Fig. 11—Plot of the times reckoning the decreasing peaks of overshoot oscillation and amplitudes of the overshoots as a function of t_r.

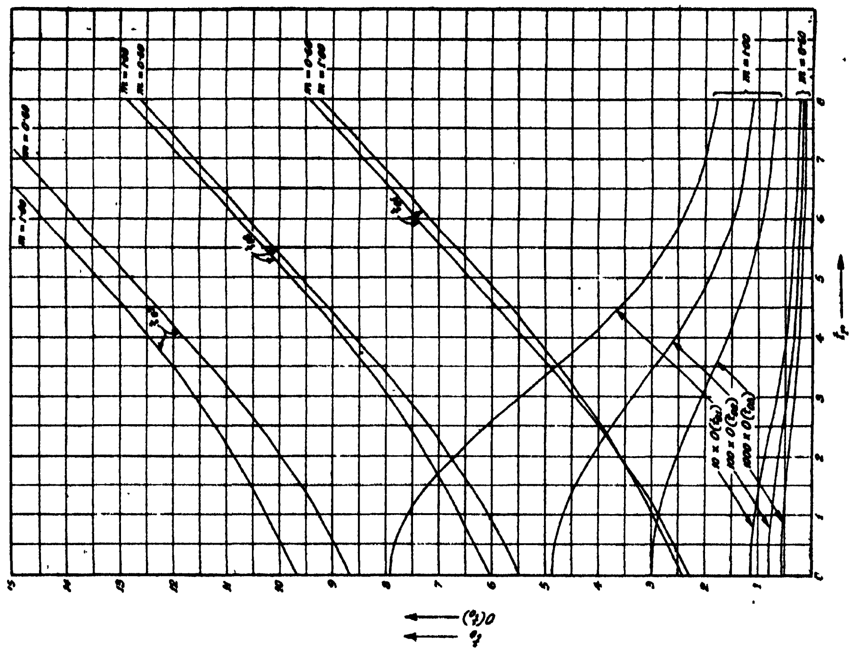

Fig. 12—Plot of the times reckoning the decreasing peaks of overshoot oscillation and amplitudes of the overshoots as a function of t_r.

A study of figures 11 and 12 will show that the overshoot of an amplifier using shunt-compensation assumes its maximum at $t_r = 0$ (a step-function input) for all possible values of m. For small values of t_r (from $t_r = 0$ to $t_r = 0.3$ approximately) the magnitude of the peak overshoot remains almost constant. As t_r increases from its value 0.3, the overshoot amplitude begins to decrease very sharply. The nature of decrease is practically exponential. The overshoot amplitudes $O(t_{01})$ for $t_r = 0.1$ and $t_r = 8.0$ are given in Table II for a better understanding of their rapid rate of fall. It may be concluded from this theoretical study that for an application where 10 per cent overshoot is not very much objectionable but the contribution of the amplifier to the rise time of leading edge of the output should be very low, the value of $m = 1.0$, for example, may be used in a specific case, $t_r = 6.0$, when $O(t_{01}) = 0.091325$ and $t_u - t_l - 0.8t_r = -0.6254$. [See next section].

TABLE II

t_r	$m = 0.35$ $O(t_{01})$	$m = 0.41$ $O(t_{01})$	$m = 0.50$ $O(t_{01})$	$m = 0.60$ $O(t_{01})$	$m = 1.00$ $O(t_{01})$
0.1	0.010034	0.029294	0.066960	0.113398	0.298315
8.0	0.001812	0.005454	0.012988	0.022883	0.066858

RISE AND DELAY TIMES OF THE OUTPUT PULSE

In the previous publication (Bhattacharyya, 1954) we have noticed that the actual contribution of the pulse amplifier to the rise and delay times of the reproduced waveforms is determined by the expressions $(t_u - t_l - 0.8t_r)$ and $(t_d - 0.5t_r)$ respectively, where t_u, t_d and t_l have been defined such that

$$e_0(t_u) = 0.9, \quad e_0(t_d) = 0.5 \text{ and } e_0(t_l) = 0.1.$$

In order to study the effect of t_r on the reproduction of the leading edge of a pulse in a shunt-compensated pulse amplifier it is now necessary to derive analytical expressions for these three parameters t_u, t_d and t_l as a function of t_r.

With the aid of (15), we can write the following equation relating t_u to t_r (when $t_u \leq t_r$) :

$$e_0(t) = \frac{1}{t_r} \cdot \frac{\epsilon^{-t/2m}}{\sqrt{4m-1}} \left[(1-m)\sqrt{4m-1} \cos\left(\frac{\sqrt{4m-1}}{2m} t\right) + (1-3m) \sin\left(\frac{\sqrt{4m-1}}{2m} t\right)\right] + \frac{1}{t_r}(t + m - 1) = \frac{9}{10} \tag{32}$$

The solution of (32) for various values of t_r will show the nature of variation of t_u with the rise time of the input pulse. This transcendental equation (32) may be solved by either graphical or numerical methods. We have used here the Newton-Raphson process (Scarborough, 1949) of finding out the roots of such equations to a sufficient degree of accuracy.

When $t_u \geq t_r$ equation (16) will have to be utilised to determine correctly the 90% points. We then have,

$$e_0'(t) = \frac{1}{t_r} \cdot \frac{\epsilon^{-t/2m}}{\sqrt{4m-1}} \left[A \cos\left(\frac{\sqrt{4m-1}}{2m} t\right) + B \sin\left(\frac{\sqrt{4m-1}}{2m} t\right)\right] + 1 = \frac{9}{10} \qquad \ldots (33)$$

Following a similar process we can now easily compute the 10% and 50% points of the response. The curves showing the rise and delay times of the output response as a function of t_r have been plotted in figures 13 and 14. The corresponding diagrams for a RC-amplifier have been included in the above figures for the purpose of visualizing the difference in response characteristics in the two cases.

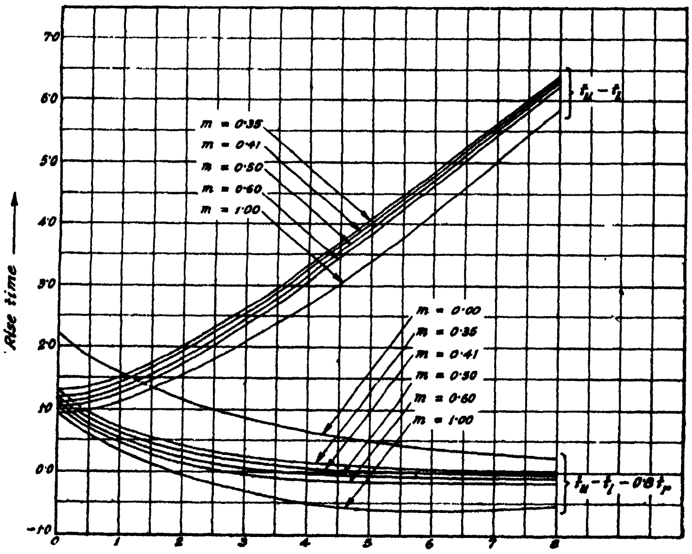

Fig. 13—Plot of time as a function of t_r.

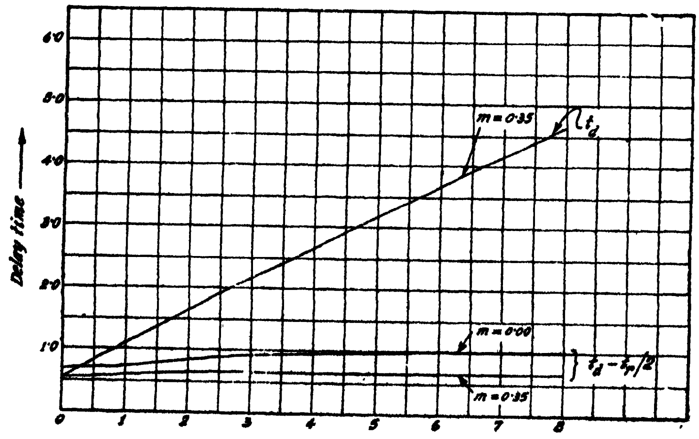

Fig. 14—Plot of delay time as a function of t_r

The contribution of the pulse amplifier to the increase of rise time of the output is significant at very small values of t_r. It reaches the maximum value for a step-function input ($t_r = 0$) and then begins to decrease at an appreciable rate with the increase of t_r. The most interesting point in the response of a shunt-compensated amplifier to such a type of input is the sharpening of the leading edge of the pulse when t_r is greater than a certain value for a specified m. Due to this reason, the curve $(t_u - t_l - 0.8t_r)$ versus t_r crosses the base line, say at $t_r = t_{rl}$. This indicates that the rise time of the reproduced pulse is less than that of the input when the magnitude of t_r surpasses the lower limit t_{rl} which is given by

$$(t_u - t_l) = 0.8t_{rl} \qquad \ldots (34)$$

Table III will give an idea about the actual measure of sharpening of the leading edge of the pulse for a particular value of t_r, equal to 8.0.

TABLE III

$(t_u - t_l - 0.8t_r)$	$m = 0.35$	$m = 0.41$	$m = 0.50$	$m = 0.60$	$m = 1.00$
(Rise time of the output)—(rise time of the input)	.0126	−.0308	−.1104	−.1821	−.5433

The delay in transmission of the input pulse by the amplifier has been shown in figure 14 as a function of t_r. Because of the very small change of the delay time t_d with the variation of m, t_d has been plotted against t_r only for the case $m = 0.35$. The values of t_d for other cases have been given in Table IV.

TABLE IV

m	$t_r = 0.1$ t_d	$t_r = 0.5$ t_d	$t_r = 1.0$ t_d	$t_r = 2.0$ t_d	$t_r = 5.0$ t_d
0.35	0.603000	0.811970	1.091636	1.652683	3.157100
0.41	0.598130	0.806574	1.082724	1.630984	3.104042
0.50	0.590000	0.797572	1.070715	1.603830	3.026846
0.60	0.583650	0.790345	1.060766	1.579841	2.946921
1.00	0.570630	0.775010	1.038642	1.522298	2.711985

With the help of this Table we may now calculate the delay introduced by the amplifier for all the interesting cases. It is noticed that with the decrease of steepness of pulse fronts the delay caused by the amplifier does not increase appreciably.

RESPONSE TO A PULSE WITH LINEAR RISE AND FALL

A pulse with linear rise and fall (figure 15) can be expressed as

$$e(t)=K\left[\frac{t}{t_1}\,u(t)-\frac{t_2}{t_1}\cdot\frac{(t-t_1)}{(t_2-t_1)}\cdot u(t-t_1)+\frac{(t-t_2)}{(t_2-t_1)}\cdot u(t-t_2)\right] \quad \ldots \quad (35)$$

where K represents the height of the pulse.

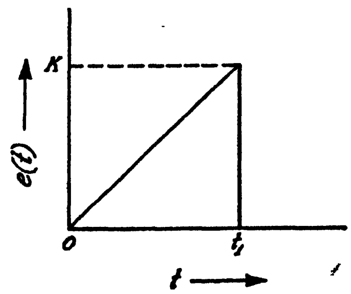

Fig. 15—A pulse with linear rise and fall.

The high-frequency equivalent circuit of the amplifier (figure 2) will only be considered. Then the normalized output voltage may be expressed in the following way :

Case I ($m<\frac{1}{4}$) :

(*i*) $\quad 0\leq t\leq t_r$

$$e_0(t)=\frac{1}{t_r}\cdot\frac{m^2}{\sqrt{1-4m}}\,\epsilon^{-t/2m}\left[\frac{1+\sqrt{1-4m}}{1-2m-\sqrt{1-4m}}\,\epsilon^{\sqrt{1-4m}\,t/2m}\right.$$

$$\left.-\frac{1-\sqrt{1-4m}}{1-2m+\sqrt{1-4m}}\cdot\,\epsilon^{-\sqrt{1-4m}\,t/2m}\right]+\frac{1}{t_r}(t+m-1), \quad \ldots \quad (36)$$

(*ii*) $\quad t_r\leq t<t_f$

$$e_0(t)=A_2\epsilon^{-(1-\sqrt{1-4m})t/2m}-A_3\epsilon^{-(1+\sqrt{1-4m})t/2m}$$

$$-\frac{t-t_f+m-1}{t_f-t_r}, \quad \ldots \quad (37)$$

(*iii*) $\quad t\geq t_f$

$$e_0(t)=C_2\epsilon^{-(1-\sqrt{1-4m})t/2m}-D_2\epsilon^{-(1+\sqrt{1-4m})t/2m} \quad \ldots \quad (38)$$

where

$$t_r=\frac{t_1}{RC},\; t_f=\frac{t_2}{RC},$$

$$A_2=\frac{m^2}{\sqrt{1-4m}}\cdot\frac{1+\sqrt{1-4m}}{1-2m-\sqrt{1-4m}}\,\frac{1}{t_r}\left[1-\frac{t_f}{(t_f-t_r)}\,\epsilon^{(1-\sqrt{1-4m})t_r/2m}\right],$$

$$A_2 = \frac{m^2}{\sqrt{1-4m}} \cdot \frac{1-\sqrt{1-4m}}{1-2m+\sqrt{1-4m}} \cdot \frac{1}{t_r}\left| 1 - \frac{t_f}{(t_f-t_r)} \cdot \epsilon^{(1+\sqrt{1-4m})t_r/2m}\right.$$

$$C_2 = \frac{m^2}{\sqrt{1-4m}} \cdot \frac{1+\sqrt{1-4m}}{1-2m-\sqrt{1-4m}} \cdot \frac{1}{t_r}\left[1 - \frac{t_f}{(t_f-t_r)} \epsilon^{(1-\sqrt{1-4m})t_r/2m}\right.$$

$$\left. + \frac{t_r}{(t_f-t_r)} \epsilon^{(1-\sqrt{1-4m})t_f/2m}\right]$$

$$D_2 = \frac{m^2}{\sqrt{1-4m}} \cdot \frac{1-\sqrt{1-4m}}{1-2m+\sqrt{1-4m}} \cdot \frac{1}{t_r}\left| 1 - \frac{t_f}{(t_f-t_r)} \cdot \epsilon^{(1+\sqrt{1-4m})t_r/2m}\right.$$

$$\left. \frac{t_r}{(t_f-t_r)} \cdot \epsilon^{(1+\sqrt{1-4m})t_f/2m}\right]$$

Case II $(m=\frac{1}{4})$

(*i*) $0 \leq t \leq t_r$

$$e_0(t) = \frac{2m}{t_r}\left[(t+6m)\epsilon^{-t/2m} + 2(t-3m) \right] \qquad \dots (39)$$

(*ii*) $t_r \leq t \leq t_f$

$$e_0(t) = \frac{2m}{t_r}(t+6m)\epsilon^{-t/2m}\left| 1 - \frac{t_f}{(t_f-t_r)}\epsilon^{t_r/2m}\right. + \frac{2mt_f}{(t_f-t_r)}\epsilon^{-(t-t_r)/2m}$$

$$+ \frac{4mt_f}{(t_f-t_r)} - \frac{4m(t-3m)}{(t_f-t_r)} \qquad \dots (40)$$

(*iii*) $t \geq t_f$

$$e_0(t) = \frac{2m}{t_r}(t+6m)\epsilon^{-t/2m}\left[1 - \frac{t_f}{(t_f-t_r)}\epsilon^{t_r/2m} + \frac{t_r}{(t_f-t_r)}\epsilon^{t_f/2m}\right]$$

$$\cdot \frac{2mt_f}{(t_f-t_r)}\epsilon^{-(t-t_r)/2m} - \frac{2mt_f}{(t_f-t_r)}\epsilon^{-(t-t_f)/2m} \qquad (41)$$

Case III $(m > \frac{1}{4})$

(*i*) $0 \leq t \leq t_r$;

$$e_0(t) = \frac{1}{t_r} \cdot \frac{\epsilon^{-t/2m}}{\sqrt{4m-1}}\left[(1-m)\sqrt{4m-1}\cos\left(\frac{\sqrt{4m-1}}{2m}t\right)\right.$$

$$\left. + (1-3m)\sin\left(\frac{\sqrt{4m-1}}{2m}t\right)\right] + \frac{1}{t_r}(t+m-1) \qquad \dots (42)$$

(*ii*) $t_r \leq t \leq t_f$

$$e_0(t) = A_1 \cos\left(\frac{\sqrt{4m-1}}{2m}t\right)\epsilon^{-t/2m} + B_1 \sin\left(\frac{\sqrt{4m-1}}{2m}t\right)\epsilon^{-t/2m}$$

$$-\frac{(t-t_f)+m-1}{(t_f-t_r)} \quad \ldots \quad (43)$$

and

(*iii*) $t \geq t_f$

$$e_0(t) = C_1\epsilon^{-t/2m} \cos\left(\frac{\sqrt{4m-1}}{2m}t\right) + D_1\epsilon^{-t/2m} \sin\left(\frac{\sqrt{4m-1}}{2m}t\right) \quad \ldots \quad (44)$$

where

$$A_1 = \frac{(1-m)}{t_r} - \frac{t_f\epsilon^{t_r/2m}}{t_r(t_f-t_r)}\left[(1-m)\cos\left(\frac{\sqrt{4m-1}}{2m}t_r\right) - \frac{(1-3m)}{\sqrt{4m-1}}\sin\left(\frac{\sqrt{4m-1}}{2m}t_r\right)\right],$$

$$B_1 = \frac{(1-3m)}{t_r\sqrt{4m-1}} - \frac{t_f\epsilon^{t_r/2m}}{t_r(t_f-t_r)}\left[(1-m)\sin\left(\frac{\sqrt{4m-1}}{2m}t_r\right) + \frac{(1-3m)}{\sqrt{4m-1}}\cos\left(\frac{\sqrt{4m-1}}{2m}t_r\right)\right],$$

$$C_1 = A_1 + \frac{\epsilon^{t_f/2m}}{(t_f-t_r)}\left[(1-m)\cos\left(\frac{\sqrt{4m-1}}{2m}t_f\right) - \frac{(1-3m)}{\sqrt{4m-1}}\sin\left(\frac{\sqrt{4m-1}}{2m}t_f\right)\right]$$

and

$$D_1 = B_1 + \frac{\epsilon^{t_f/2m}}{(t_f-t_r)}\left[(1-m)\sin\left(\frac{\sqrt{4m-1}}{2m}t_f\right) + \frac{(1-3m)}{\sqrt{4m-1}}\cos\left(\frac{\sqrt{4m-1}}{2m}t_f\right)\right]$$

We shall consider here only the last case ($m > \frac{1}{4}$). With the help of equations (42—44) the response functions have been plotted in figures 16-20.

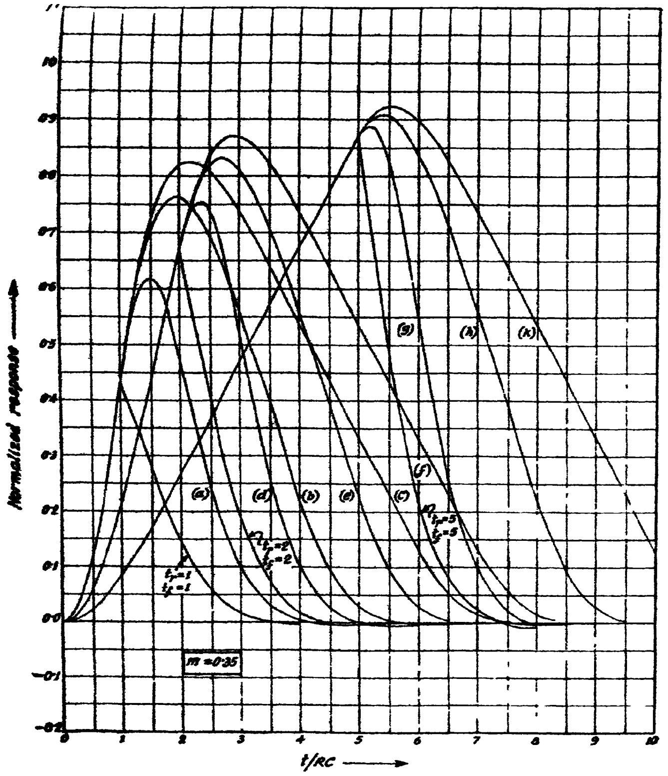

Fig. 16—Response of a shunt-compensated amplifier to a pulse with linear rise and fall ($m = 0.35$):

(a) $t_r = 1$, $t_f = 2$; (b) $t_r = 1$, $t_f = 4$;
(c) $t_r = 1$, $t_f = 6$; (d) $t_r = 2$, $t_f = 3$;
(e) $t_r = 2$, $t_f = 5$; (f) $t_r = 2$, $t_f = 7$;
(g) $t_r = 5$, $t_f = 6$, (h) $t_r = 5$, $t_f = 8$,
(k) $t_r = 5$, $t_f = 10$.

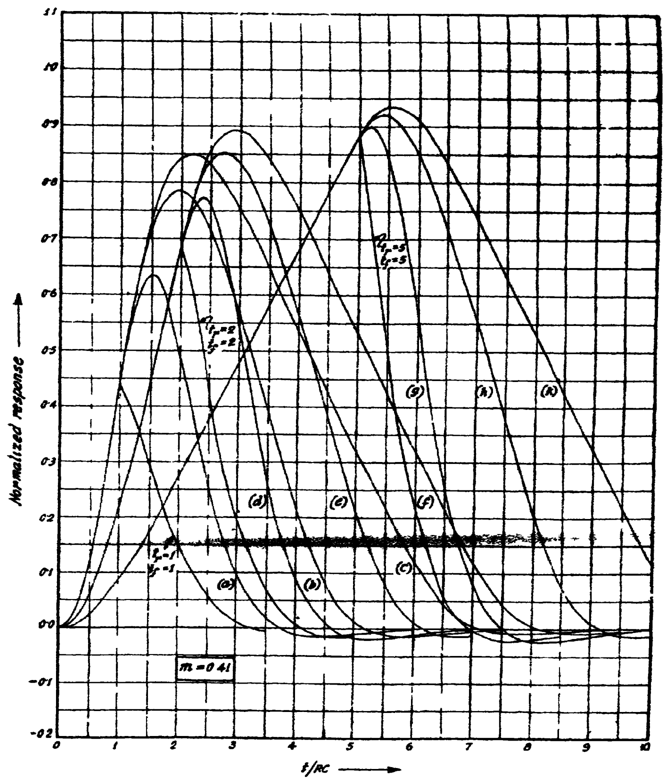

Fig. 17—Response of a shunt-compensated amplifier to a pulse with linear rise and fall ($m = 0.41$).

(a) $t_r = 1$, $t_f = 2$; (b) $t_r = 1$, $t_f = 4$,
(c) $t_r = 1$, $t_f = 6$, (d) $t_r = 2$, $t_f = 3$,
(e) $t_r = 2$, $t_f = 5$; (f) $t_r = 2$ $t_f = 7$,
(g) $t_r = 5$, $t_f = 6$, (h) $t_r = 5$, $t_f = 8$,
(k) $t_r = 5$, $t_f = 10$.

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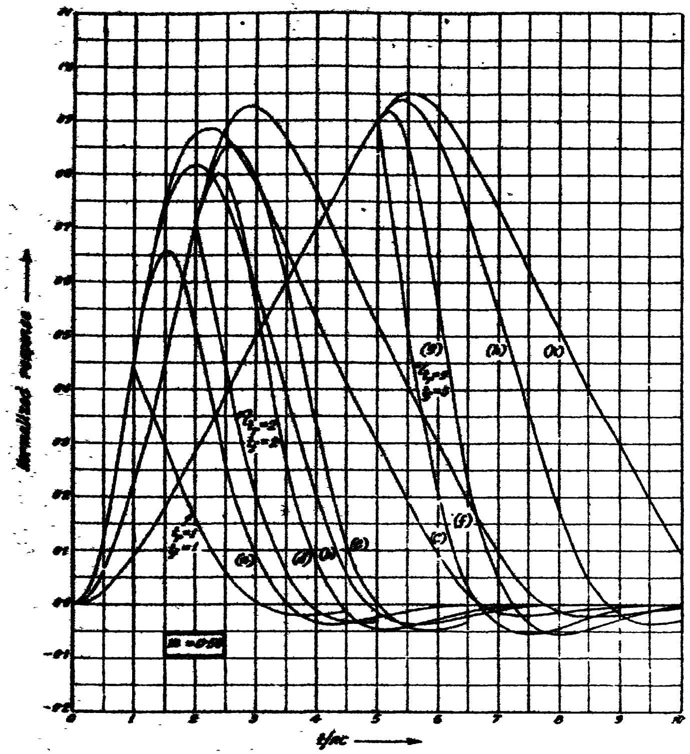

Fig. 18—Response of a shunt-compensated amplifier to a pulse with linear rise and fall (m = 0.50) :
(a) $t_r=1,\ t_f=2$; (b) $t_r=1,\ t_f=4$;
(c) $t_r=1,\ t_f=6$; (d) $t_r=2,\ t_f=3$;
(e) $t_r=2,\ t_f=4$; (f) $t_r=2\ \ t_f=7$;
(g) $t_r=5,\ t_f=6$; (h) $t_r=5,\ t_f=8$;
(k) $t_r=5,\ t_r=10$.

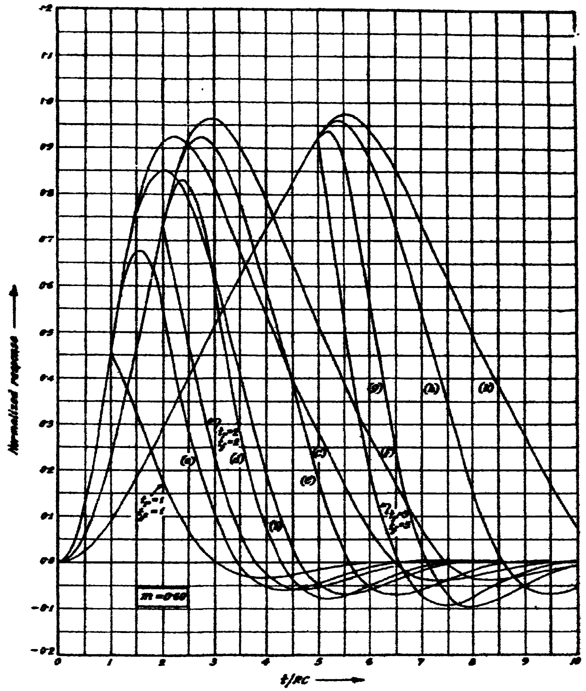

Fig. 19—Response of a shunt compensated amplifier to a pulse with linear rise and fall (m = 0.60) :
(a) $t_r=1,\ t_f=2$; (b) $t_r=1,\ t_f=4$;
(c) $t_r=1,\ t_f=6$; (d) $t_r=2,\ t_f=3$;
(e) $t_r=2,\ t_f=5$; (f) $t_r=2,\ t_f=7$;
(g) $t_f=5,\ t_f=6$; (h) $t_r=5,\ t_f=8$;
(k) $t_r=5,\ t_f=10$.

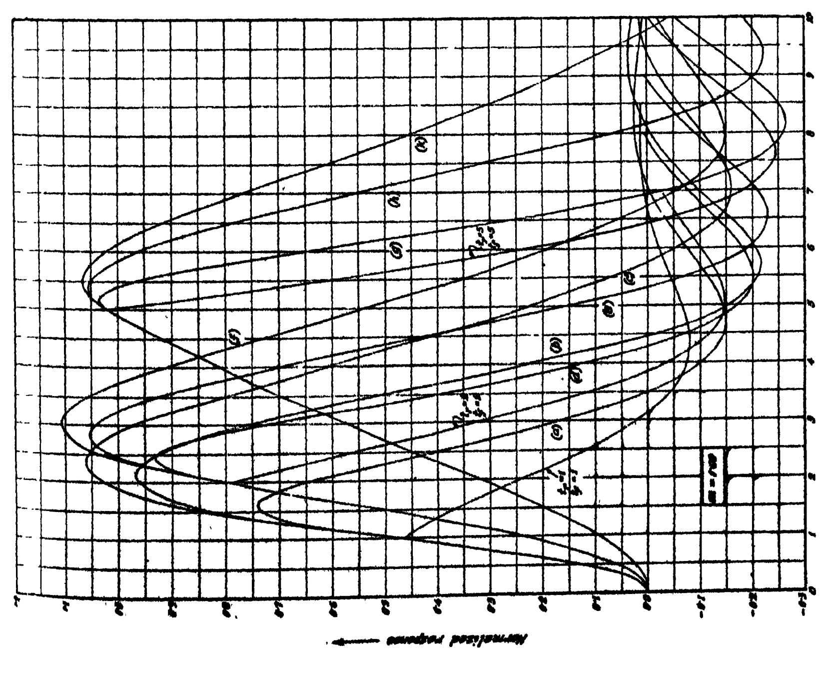

Fig. -Response of a shunt compensated amplifier to pulse with linear rise and fall (m = 1.00):

(a) $t_r=1$, $t_f=2$; (b) $t_r=1$, $t_f=4$;
(c) $t_r=1$, $t_f=6$; (d) $t_r=2$, $t_f=3$;
(e) $t_r=2$, $t_f=5$; (f) $t_r=2$, $t_f=7$;
(g) $t_r=5$, $t_f=6$; (h) $t_r=5$, $t_f=8$;
(k) $t_r=5$, $t_f=10$

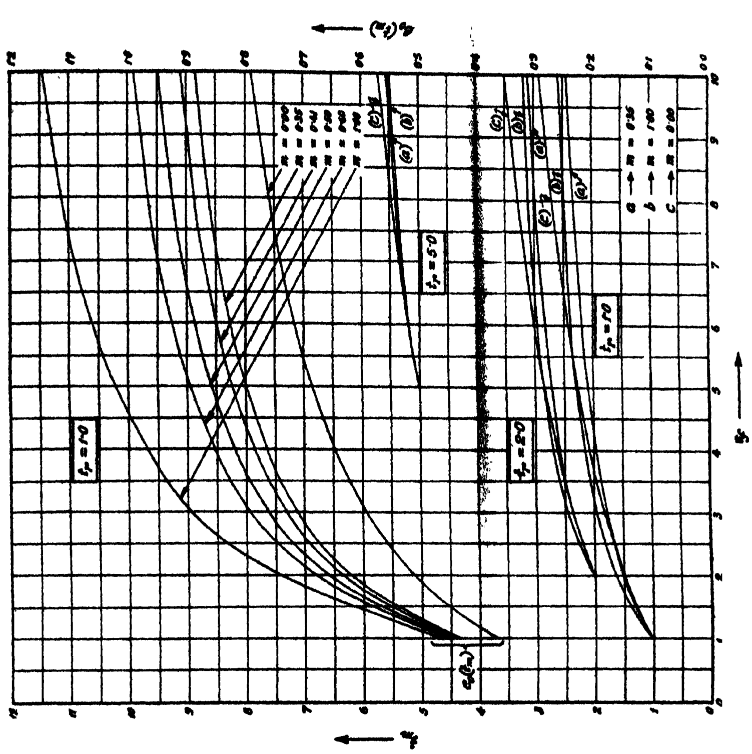

Fig. 21—A plot of t_m and $e_o(t_m)$ against t_f for different values of t_r.

It is noticed from the figures that the maximum amplitude gradually increases as the steepness of fall is reduced for a constant time of rise. The maximum amplitude occurs at a time t_m which lies within the interval $t_r \leq t_m \leq t_f$. So by differentiating the expression for the output voltage $e_0'(t)$ (equation 43) and equating $de_0(t)/dt$ to zero we can obtain an equation the root of which is equal to t_m. The equation is given below :

$$\epsilon^{-t/2m}\cos\left(\frac{\sqrt{4m-1}}{2m}t\right)\left[B_1\frac{\sqrt{4m-1}}{2m}-\frac{A_1}{2m}\right]$$

$$-\epsilon^{-t/2m}\sin\left(\frac{\sqrt{4m-1}}{2m}t\right)\left[A_1\frac{\sqrt{4m-1}}{2m}+\frac{B_1}{2m}\right]=\frac{1}{(t_f-t_r)} \quad \cdots \quad (45)$$

The values of t_m obtained by solving (45) are substituted in (43) to find out the magnitude of the maximum amplitude for particular values of t_r and t_f.

A plot of t_m against t_f for different values of t_r is given in figure 21. Only two values of m, viz. (i) $m=0.35$ and (ii) $m=1.00$ have been considered because of the very small change of t_m with m. Curves of the maximum amplitude $e_0(t_m)$ have been drawn against t_f in figures 21 – 23 for three values of t_r viz. (i) $t_r=1.0$, (ii) $t_r=2.0$ and (iii) $t_r=5.0$. For the sake of comparison, the maximum amplitude curves of ordinary RC-coupled amplifiers have also been included in every figure. It is noticed that the peak of the output voltage of a shunt-compensated amplifier is considerably greater than that of an RC-coupled amplifier.

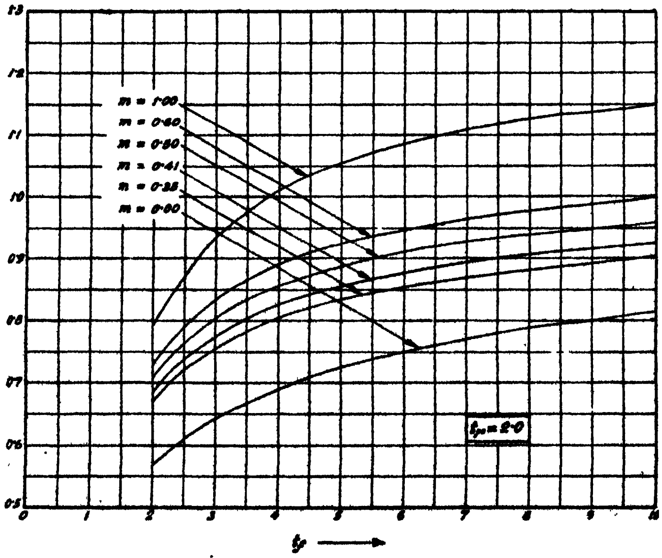

Fig. 22—A plot of $e_0(t_m)$ against t_f ($t_r=2.0$).

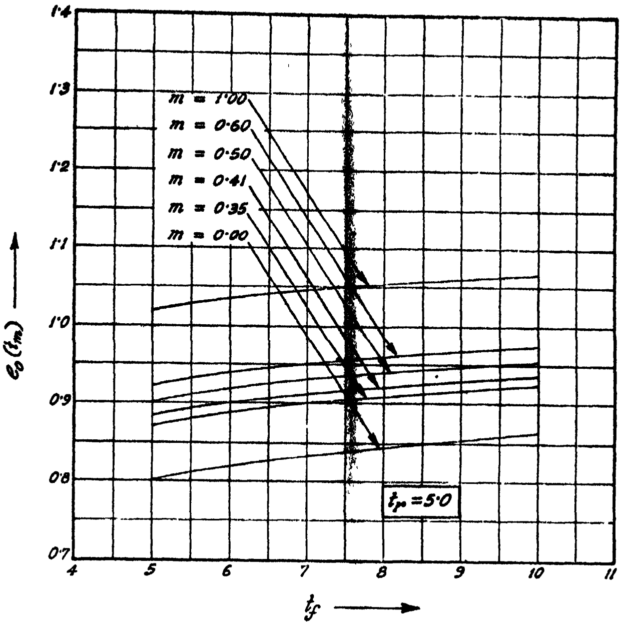

Fig. 23—A plot of $e_o(t_m)$ against t_f ($t_r = 5.0$).

Figures 16 – 20 also indicate that the amplitudes of the undershoots are appreciable when $m > 0.50$. Undershoots reach the peak value at times $t_0 > t_f$. These points may be accurately determined by solving the following equation which is obtained by differentiating (44)

$$\left[\frac{D_1}{2m} + C_1\frac{\sqrt{4m-1}}{2m}\right] \sin \frac{\sqrt{4m-1}}{2m} t$$

$$- \left[\frac{C_1}{2m} + D_1\frac{\sqrt{4m-1}}{2m}\right] \cos \left(\frac{\sqrt{4m-1}}{2m} t\right) \qquad (46)$$

The times t_0 thus determined are then substituted in (44) to obtain the peak magnitude of the undershoot. The nature of variation of these peak values with change in t_f has been shown in figure 24. The other

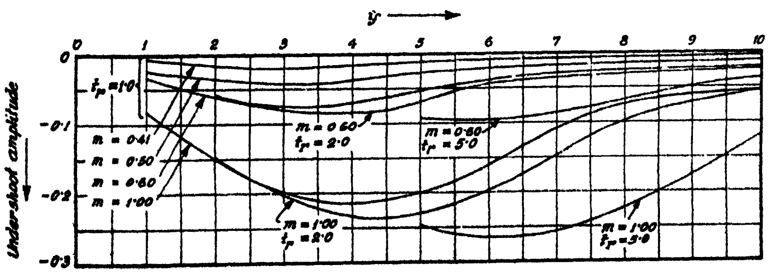

Fig. 24—A plot of undershoot amplitudes against t_f.

charcteristics of the undershoot oscillation are similar to those observed in the case of the overshoots of a shunt-compensated amplifier for a ramp function input and may be determined in the same way as outlined previously.

RESPONSE TO A SAW-TOOTH PULSE

When $t_f = t_r$, the pulse shown in figure 15 takes the shape of a saw-tooth waveform and may be represented by

$$e(t) = K\frac{t}{t_1}\,[u(t) - u(t-t_1)] \qquad \ldots (47)$$

In this case the normalized response functions are given by the following expression ($m > \frac{1}{4}$) :

$$e_0(t) = \frac{1}{t_r}\cdot\frac{\epsilon^{-t/2m}}{\sqrt{4m-1}}\left[(1-m)\sqrt{4m-1}\,\cos\left(\frac{\sqrt{4m-1}}{2m}t\right)\right.$$
$$\left.+(1-3m)\sin\left(\frac{\sqrt{4m-1}}{2m}t\right)\right] + \frac{1}{t_r}(t+m-1),\quad 0 \leq t \leq t_r \qquad \ldots (48)$$

and

$$e_0(t) = \frac{1}{t_r}\cdot\frac{\epsilon^{-(t-t_r)/2m}}{\sqrt{4m-1}}\left[A_0\cos\left(\frac{\sqrt{4m-1}}{2m}t\right) + B_0\sin\left(\frac{\sqrt{4m-1}}{2m}t\right)\right]$$
$$+\epsilon^{-(t-t_r)/2m}\left[\cos\left\{\frac{\sqrt{4m-1}}{2m}(t-t_r)\right\} - \frac{(1-2m)}{\sqrt{4m-1}}\sin\left\{\frac{\sqrt{4m-1}}{2m}(t-t_r)\right\}\right],$$
$$t \geq t_r \qquad \ldots (49)$$

where

$$A_0 = (1-m)\sqrt{4m-1}\left[\epsilon^{-t_r/2m} - \cos\left(\frac{\sqrt{4m-1}}{2m}t_r\right)\right] + (1-3m)\sin\left(\frac{\sqrt{4m-1}}{2m}t_r\right)$$

and

$$B_0 = (1-3m)\left[\epsilon^{t_r/2m} - \cos\left(\frac{\sqrt{4m-1}}{2m}t_r\right)\right] - (1-m)\sqrt{4m-1}\,\sin\left(\frac{\sqrt{4m-1}}{2m}t_r\right)$$

The expressions of $e_0(t)$ for the other two cases, viz. (*i*) $m=\frac{1}{4}$ and (*ii*) $m<\frac{1}{4}$, will not be considered here. With the aid of equations (48) and (49) the response characteristics in the case of a saw-tooth input have been plotted in figures 16 – 20. It is observed from the figures that the maximum amplitude occurs at a time $t = t_f$ and this assumes a much greater magnitude when shunt-compensation is used in an RC-coupled amplifier.

CONCLUSION

This investigation of the effect of ramp function input on the response characteristics of shunt-compensated pulse amplifiers leads us to some interesting conclusions.

First of all, when the input pulse is not very sharp, the magnitude of peak overshoots may be made negligible for many practical applications by a suitable choice of m. For example, let us consider the case of $t_1 = 3.5\ RC$, i.e., $t_r = 3.5$ and $m = 0.5$. Then the overshoot reaches its peak value .030854 (3 percent of the normalized value) at the time $t_{01} = 5.060497$.

Secondly, the contribution of the pulse amplifier to the rise time of the output is not at all appreciable when the build-up time of the input is not very short. If we consider the previous case, we find that $(t_u - t_l - 0.8t_r)$ is only -0.008097. We, therefore, find that when the amplifier RC constant is several times smaller than the input pulse rise time, the transmission of the leading edge of the pulse through the amplifier may be made very faithful.

An unexpected result of this analysis is the finding that when the rise time of the input pulse becomes greater by several times the RC constant of the plate circuit, the output pulse may be sharper compared to the input for values of m higher than a lower limit which is determined by the rise time of the incoming wavefront. In the case of pulses with linear rise and fall, it has been noticed that the maximum amplitude of the output pulse and the time that corresponds to this maximum, depend very much upon times of both rise and fall of the input. The maximum amplitude of the output increases noticeably when shunt-compensation is used in an RC-coupled amplifier.

This study of pulse amplifiers using shunt compensation gives us important design information which cannot be obtained from the step-function response characteristics (Valley and Wallman, 1948) It was, therefore, felt necessary to derive the response functions of pulse amplifiers with the assumption of an input waveform the shape of which is the best approximation to that of the actual input.

ACKNOWLEDGMENTS

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